THE ONE-SAMPLE K-S TEST: TI-59 PROGRAMS FOR DISCRETE AND CONTINUOUS RANDOM VARIABLES

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THESIS

THE ONE-SAMPLE K-S TEST: TI-59 PROGRAMS
FOR DISCRETE AND CONTINUOUS RANDOM VARIABLES

by

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for Discrete and Continuous Random Variables

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ABSTRACT

The Kolmogorov-Smirnov (K-S) goodness-of-fit test is a nonparametric test if the random variable is continuous. If the random variable is discrete the K-S test is not nonparametric.

Two programs for the TI-59 programmable calculator are presented, "Continuous K-S" and "Discrete K-S". They are designed to conduct a K-S test and display the significance level. User instructions and several sample problems are presented.

Use of the "hidden registers" in the TI-59 is discussed. The hidden registers are used extensively in the program Discrete K-S.

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I. INTRODUCTION

The subject of this thesis is the one-sample Kolmogorov-Smirnov (K-S) goodness-of-fit test. The purpose of the thesis is to provide the reader with programs enabling him to perform K-S tests on the TI-59 programmable calculator.

The distinction between continuous and discrete random variables is a central issue. Different computational procedures are appropriate when testing a discrete random variable than when testing a continuous random variable. Two separate TI-59 programs are provided: "Continuous K-S" and "Discrete K-S".

Section II is a general discussion of the K-S test. Section III explains the technique of transforming sample data before performing a K-S test. Section IV discusses the distribution of test statistics from a continuous K-S test and provides detailed instructions on how to use the Continuous K-S program. Section V is an analagous discussion for a discrete K-S test.

Appendicies A and B are entry instuctions, program listings, and example problems for Continuous K-S and Discrete K-S, respectively. Appendix C presents condensed user instructions for both programs. Appendix D discusses the use of the "hidden registers" in the TI-59.

II. THE KOLMOGOROV-SMIRNOV (K-S) TEST

A. OVERVIEW OF THE K-S TEST

Statisticians are often faced with observations of a random variable based on a population with an unknown probability distribution. A goodness-of-fit test may be used to test certain hypotheses about the relationship between the population's distribution and some specified distribution. The one-sample Kolmogorov-Smirnov (K-S) test is a goodness-of-fit test. The term "one-sample" is used because a "two-sample" K-S test has also been developed. The two-sample test examines the relationship between two unknown probability distributions on the basis of two random samples from different populations. Only the one-sample K-S test is discussed herein. It will be refered to below as simply the "K-S test".

The K-S test is conducted by comparing S_n, the empirical cumulative distribution function (CDF), defined below, with H, the hypothesized CDF. If the discrepancy between S_n and H, called the "test statistic", is large then we have strong evidence that F, the population CDF, does not equal H. The probability of observing a test statistic at least as large as that realized is called the "significance level" of the test.

A K-S test is only valid if outcomes on the random variable lie on an ordinal scale. If a random variable maps events into cells between which no ordinal relationship exists then the chi square goodness-of-test is recommended [Ref. 1: pp. 237].

Any goodness-of fit test can be either a hypothesis test or a significance test. For a hypothesis test we specify a null hypothesis, H_0, an alternative hypothesis, H_1, and an "alpha level". After we compute the significance level we compare it with the alpha level. If the significance level is less than or equal to the alpha level then we reject H_0 and accept H_1. Otherwise, we do not reject H_0. To conduct a significance test we simply compute the significance level; we are not formally concerned about rejecting H_0.

The requisite computational techniques, yet to be discussed, are identical for hypothesis and significance testing. For either type of test the test statistic must be computed and then evaluated by computing the significance level. Hence, the discussion of the K-S test below will focus on those two steps.

Let X_1 , X_2, . . . , X_n be independent observations on the random variable X which has CDF F. Assume an ordinal relationship exists between all possible values of X. If $X_{(1)}$, $X_{(2)}$, . . . $X_{(n)}$ represent the n observations arranged in ascending order then the empirical CDF, S_n is defined as:

$$S_n(x) = \begin{cases} 0 & \text{if} \quad x < X_{(1)} \\ k/n & \text{if} \quad X_{(k)} \leq x < X_{(k+1)}, \quad k = 1, 2, \ldots, n-1 \\ 1 & \text{if} \quad x \geq X_{(n)} \end{cases}$$

The K-S test may be used to test the following three pairs of hypotheses given a specified CDF H. The appropriate test statistic is also listed.

1. H_0: $F(x) \geq H(x)$ for all x

   H_1: $F(x) < H(x)$ for some x

   Test statistic: $D^- = \sup_x(H(x) - S_n(x))$

2. H_0: $F(x) \leq H(x)$ for all x

   H_1: $F(x) > H(x)$ for some x

   Test statistic: $D^+ = \sup_x(S_n(x) - H(x))$

3. H_0: $F(x) = H(x)$ for all x

   H_1: $F(x) \neq H(x)$ for some x .

   Test statistic: $D = \max(D^-, D^+)$

Note that by simply specifying H_0 we implicitly specify H_1 since H_1 is the negation of H_0. It should also be noted that the first two null hypotheses listed pertain to what is commonly called "one-sided" tests while the third pertains to a "two-sided" test. The test statistics, D^-. D^+, and D, are collectively called the "K-S statistics".

B. CALCULATING THE VALUE OF THE TEST STATISTIC

1. Discrete Random Variable

When examining a discrete random variable the the test statistics D^- and D^+ are found by comparing the

empirical and hypothesized CDFs at each possible value of the random variable. More precisely,

$$D^{+} = \max \{0, \max[S_n(x) - H(x)]\}$$

and $$D^{-} = \max \{0, \max[H(x) - S_n(x)]\} .$$

Figure 1 illustrates this approach.

2. Continuous Random Variable

If we are examining a continuous random variable we calculate D^{+} and D^{-} using the values of the empirical and hypothesized CDFs at the observed values in our sample. The computation of D^{+} is analagous to the procedure used with a discrete random variable. The computation of D^{-}, however, is conceptually different. Using previous notation:

$$D^{+} = \max\{0, \max[S_n(X_{(i)}) - H(X_{(i)})]\}, \quad i = 1, 2, \ldots, n$$

$$D^{-} = \max\{0, \max[H(X_{(i)}) - S_n(X_{(i-1)})]\}, \quad i = 1, 2, \ldots, n$$

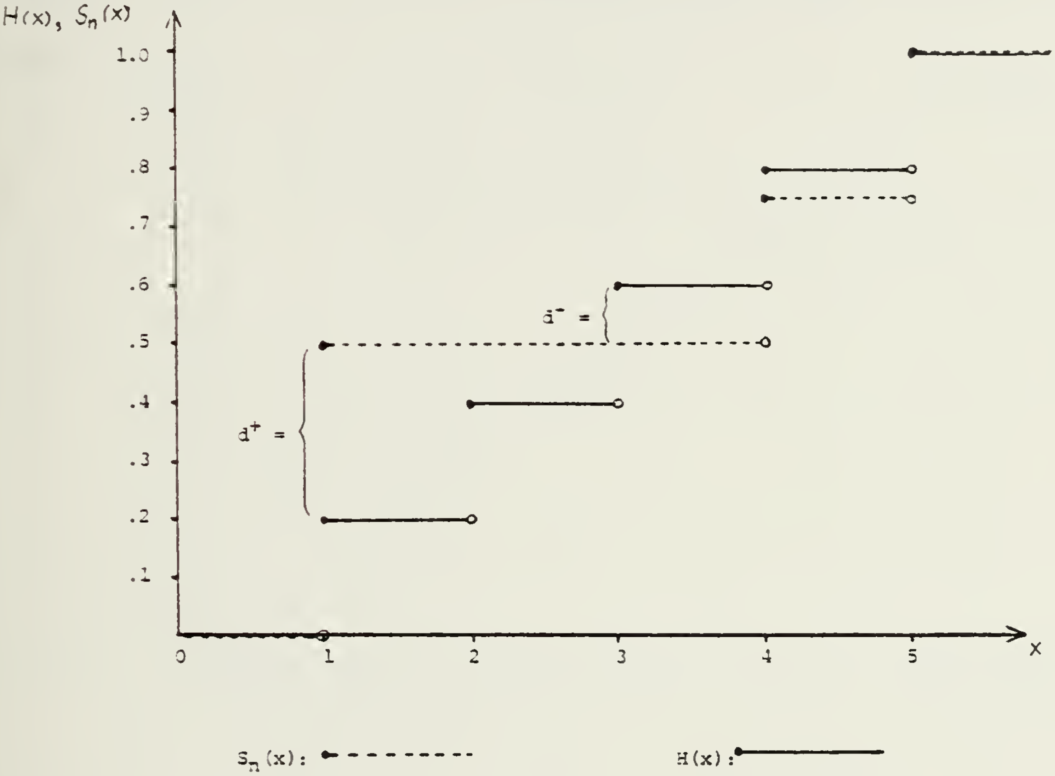
We define $X_{(0)} = -\infty$, thus $S_n(X_{(0)}) = 0$. Figure 2 illustrates this approach.

C. HISTORICAL ORIGINS OF THE K-S TEST

The K-S test was first introduced by Kolmogorov in 1933 [Ref. 2] and was further developed by Smirnov [Ref. 3]. The essence of the original work in this field was the derivation of the distribution of the K-S statistics D^{+}, D^{-}, and D. Continuity of the random variable was a key assumption in the development of the test. The statistics D^{+} and D^{-} were discovered to follow the same distribution and this distribution was derived exactly. The distribution

Hypothesized distribution: $H(x) = x/5$, $x = 1, 2, 3, 4, 5$

Sample data: 1, 1, 4, 5 (sample size $n = 4$)



$$
\begin{aligned}
d^{+} &= \max\{\emptyset, \max_{x}[S_n(x) - H(x)]\}, \quad x = 1, 2, 3, 4, 5 \\
&= \max\{\emptyset, \max[(.5-.2), (.5-.4), (.5-.6), (.75-.8), (1-1)]\} \\
&= \max\{\emptyset, \max[.3, .1, -.1, -.\emptyset5, \emptyset]\} \\
&= .3
\end{aligned}
$$

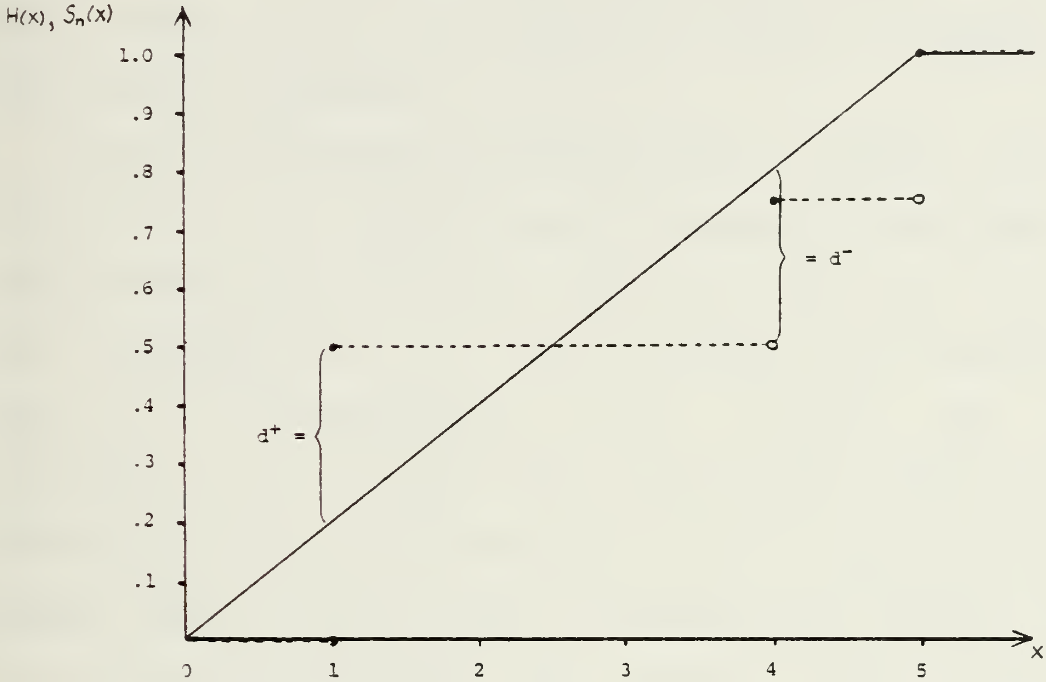
$$
\begin{aligned}
d^{-} &= \max\{\emptyset, \max_{x}[H(x) - S_n(x)]\}, \quad x = 1, 2, 3, 4, 5 \\
&= \max\{\emptyset, \max[(.2-.5), (.4-.5), (.6-.5), (.8-.75), (1-1)]\} \\
&= \max\{\emptyset, \max[-.3, -.1, .1, .\emptyset5, \emptyset]\} \\
&= .1
\end{aligned}
$$

Figure 1

Example: Test Statistics from a Discrete K-S

Hypothesized distribution: $H(x) = \begin{cases} 0, & x < 0 \\ x/5, & 0 \leq x \leq 5 \\ 1, & x > 5 \end{cases}$

Sample data: 1, 1, 4, 5 (ties occur from rounding error)



$S_n(x)$: •- - - - - - - - - $H(x)$: •————

$$\begin{aligned} d^+ &= \max\{0, \max_i [S_n(x_{(i)}) - H(x_{(i)})]\}, \quad i = 1, 2, 3, 4 \\ &= \max\{0, \max[(.5-.2), (.5-.2), (.75-.8), (1-1)]\} \\ &= \max\{0, \max[.3, .3, -.05, 0]\} \\ &= .3 \end{aligned}$$

$$\begin{aligned} d^- &= \max\{0, \max_i [H(x_{(i)}) - S(x_{(i-1)})]\}, \quad i = 1, 2, 3, 4 \\ &= \max\{0, \max[(.2-0), (.2-.5), (.8-.5), (1-.75)]\} \\ &= \max\{0, \max[.2, -.3, .3, .25]\} \\ &= .3 \end{aligned}$$

note: by definition, $x_{(\phi)} = -\infty$, thus, $S_n(x_{(\phi)}) = 0$

Figure 2

Example: Test Statistics from a Continuous K-S

of D was closely approximated. The accuracy of the approximation was shown to be very high when the significance level of a test is small. Bradley [Ref. 4: pp. 296] has presented a derivation of the distribution of the K-S statistics which is based upon, but more readable than, the original arguments.

When the hypotheszed distribution, H, equals the true distribution, F, and the random variable is continuous then the distributions of the K-S statistics have been shown to be independent of H. For this reason the continuous K-S test is a "nonparametric" test. Tables are widely available, e.g., Bradley [Ref. 4: pp. 367], delineating the distribution of the K-S statistics as a function only of sample size. This is done by showing the "critical values" of the test statistics for certain widely used alpha levels. The user is thus relieved of having to compute the significance level when performing a hypothesis test. He need only compute the test statistic and compare it to the critical value.

It is incorrect to consider the K-S test to be a nonparametric test when the random variable of interest is discrete. Conover [Ref. 5] has shown how the distributions of the K-S statistics for a discrete random variable are dependent upon both the particular hypothesized distribution and the sample size. If the K-S statistics which result from testing a discrete random variable are assumed to follow the

distributions derived by Kolmogorov then the significance level computed will be larger than the true significance level. Similarly, reliance on tables such as Bradley's will impose critical values that are unnecessarily large for a given alpha level. By either relying directly upon Kolmogorov's distributions or upon tables such as Bradley's the effect is the same. The evidence against H_o, the null hypothesis, will appear much weaker (or "less significant") than it really is.

III. TRANSFORMATION OF SAMPLE DATA

The K-S test is usually applied without employing transformations on the sample data. However, since it is sometimes convenient to transform data before conducting a K-S test, the Continuous K-S program is desgned to utilize transformations as a user option. Transformations are not recommended for a discrete K-S test.

Assume W is a continuous random variable with CDF H and a transformation M is defined such that M(W) has a known CDF, G. Further assume that we have observations on a continuous random variable X with some unknown CDF, F. We may conduct a K-S test of our hypothesis concerning the relationship between F and H in two ways. Without using any transformation the empirical CDF S_n may be compared to H. Alternatively, the empirical CDF S_n' may be compared to G, where S_n' is based upon observations of the random variable M(X). Both methods will yield the same test statistic and significance level.

A frequent use of this technique is to define the transformation M to be the function H, i.e., to invoke the probability integral transformation [Ref. 6: pp. 202]. For instance, given a continuous random variable X with CDF H the probability integral transformation tells us that the random variable H(X) is distributed uniform (0,1). If X is

the population random variable in a K-S test and H is the hypothesized CDF, we may wish to compare the empirical CDF S_n' from observations on H(X) to a uniform (0,1) CDF. This might be easier than directly comparing S_n to H. Example 3 in Appendix A illustrates this approach.

As another example, it is sometimes convenient to employ a transformation when the hypothesized distribution is lognormal. By definition, the random variable W is distributed lognormal (μ, σ^2) if and only if ln(W) is distributed normal (μ, σ^2) [Ref. 6: pp. 117]. Thus, if we have observations on a random variable X and we hypothesize that X is distributed lognormal (μ, σ^2), we may simplify computation by transforming the data. By treating the values of ln(x) as our sample data and normal (μ, σ^2) as our hypothesized distribution we can take advantage of the normal CDF subroutine in the TI-59 calculator. See example 4 in Appendix A.

IV. THE K-S TEST: CONTINUOUS RANDOM VARIABLE

A. DISTRIBUTIONS OF THE TEST STATISTICS

The test statistic in a K-S test is a random variable with a particular probability distribution. After conducting a K-S test with test statistic T ($T = D^+$, D^-, or D) we observe some value, t, from the distribution of T. To find the test's significance level we compute $P(T \geq t)$. If $T = D^+$ or D^- it was shown by Birnbaum and Tingey [Ref. 7] that

$$\text{(IV.1)} \qquad P(T \geq t) = t \sum_{j=0}^{[n(1-t)]} \binom{n}{j} (1-t-j/n)^{n-j} (t+j/n)^{j-1}$$

where n is sample size, $\binom{n}{j} = n!/j!(n-j)!$, and $[n(1-t)]$ is the largest integer contained in $n(1-t)$. For $T = D$ the probability value above should be doubled. The significance levels computed with equation IV.1 will be exact for one-sided tests. For a two-sided test the doubled result will be an upper bound on the true significance level. The difference between the upper bound and the exact value will be negligible when $P(D \geq t)$ is small.

For large sample sizes there are useful approximations to equation IV.1, but their accuracies are marginal for sample sizes as small as $n = 50$. On the other hand, the

difficulty in using equation IV.1 for hand computation is considerable for sample sizes greater then n = 10. Furthermore, even finding the value of the test statistic can be tedious. The Continuous K-S program implements equation IV.1 using a TI-59 calculator and is listed in Appendix A along with several example problems. The program is designed for sample sizes less than n = 55 and is used as follows.

B. USER INTRUCTIONS FOR "CONTINUOUS K-S" PROGRAM

STEP ONE - Specifying the test: The user must choose which continuous probability distribution, H, will be compared against the empirical distribution from the sample data (or the transformed sample data). The program's options for H are: normal, exponential, Weibull, or uniform. If H is normal then the Applied Statistics module must be used, otherwise, any module may be used in the calulator. The user must specify the appropriate parameter(s) of H.

STEP TWO - Entering the data: Once the test is specified, read both sides of Card 1 (Continuous K-S) into the TI-59. A printer may be used but it is not required. To enter the data (in any order):

Enter	Press	Display	Printer (Optional)
	E	399.69	
X_i ($i = 1, \ldots, n$)	A	i	X_i
	C	1	"INPUT DATA"

$$
\begin{array}{l}
\text{sorted data} \left\{ \begin{array}{ll} X_{(1)} & 01 \\ X_{(2)} & 02 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ X_{(n)} & n \end{array} \right. \\
\text{ignore} \left\{ \begin{array}{ll} 0 & n+1 \\ 0 & n+2 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{array} \right.
\end{array}
$$

There will be a delay after pressing "C" while the calculator sorts the data into ascending order. For a sample of size 10 this will take about 1 minute while for the maximum sample size of 54 the sorting requires about 15 minutes. The user may find it preferable to sort the data himself. If the user enters the data in ascending order the delay will be between 10 and 75 seconds after pressing "C", depending upon sample size. If the printer is used a listing of the ordered input data will be produced.

If the sorted data are to be tested directly without the use of a transformation the next step should be omitted.

STEP THREE - Transforming the data: If a transormation M is desired the user must provide it as a subroutine for Card 1. The subroutine must begin with "LBL E′" and end with "INV SBR". After Card 1 has been read into the

calculator the user should enter the subroutine, beginning at program step 325. It must not exceed 75 steps in length. For a sample size of size n, the subroutine must use only registers numbered n+1 through 57. The subroutine should be written to be called when value X is currently in the "x-register" and M(X), i.e., the transformed value, is to be returned to the main program by the subroutine.

For example, assume a transformation t defined as :

$$M(X) = (X-2)/3 .$$

After reading Card 1 into the calculator the appropriate steps are:

Press	Display
GTO 325	
LRN	325 00
LBL	326 00
E′	327 00
−	328 00
2	329 00
)	330 00
÷	331 00
3	332 00
)	333 00
INV SBR	334 00
LRN	

After entering the tranformation the user should test it before proceeding. In the above example M(8) = 2. To test: enter "8", press "E′", and the calculator should display "2".

After validating the transformation subroutine the user should press "B′". When then transformation is completed $M(X_{(n)})$ is displayed. If a printer is used the transformed

data are listed. The user is now ready to perform the K-S test.

If the hypothesized distribution, H, is lognormal (μ, σ^2) then a different procedure is followed since the appropriate transformation is provided as an optional subroutine on Card 1. After entering the data and pressing "C" the program will sort (or confirm that the user has sorted) the original data. The user then presses "A′". This will transform each observed value of X to ln(X). The calculator will display $\ln(X_{(n)})$ a few moments later and, if a printer is used, a listing of the transormed data will be produced. The K-S test can then be conducted by comparing the transformed data to a normal (μ, σ^2) distribution.

If H is lognormal then a K-S test can be performed with Continuous K-S only if the sample data is transformed. A lognormal random variable is strictly positive. Hence, if any input values are nonpositive the calculator will flash after "A′" is pressed, indicating an error.

STEP FOUR - Conducting the test: Both sides of Card 2 (Continuous K-S) should be read into the calculator. Card 2 is initialized differently depending on the hypothesis being tested.

a. Initialization:

(1) press A if H_0 : $F(x) = H(x)$ for all x
H_1 : $F(x) \neq H(x)$ for some x

(2) press B if H_0 : $F(x) \geq H(x)$ for all x
H_1 : $F(x) < H(x)$ for some x

(3) press C if H_0 : $F(x) \leq H(x)$ for all x
H_1 : $F(x) > H(x)$ for some x

In all cases "0" is displayed. If the user changes his mind about the direction of his hypotheses after initializing he should press "RST" before pressing the correct user label.

b. Defining H and computing significance level (S.L.):

(1) Normal (μ, σ^2)

Enter	Press	Display
μ	A′	μ
σ	R/S	S.L.

(2) Exponential (λ) $\{ H(x) = 1 - e^{-\lambda x} \; ; \; x \geq 0 \}$

Enter	Press	Display
λ	B′	1
1	R/S	S.L.

(3) Weibull (λ, α) $\{ H(x) = 1 - e^{-(\lambda x)^{\alpha}} \; ; \; x \geq 0 \}$

Enter	Press	Display
λ	B′	1
α	R/S	S.L.

(4) Uniform (a,b)

Enter	Press	Display
a	C′	a
b	R/S	S.L.

The computation time will vary from 1 minute with sample size of 10 up to 20 minutes for the maximum sample size of 54. After the computation is finished and the significance level is displayed the user may recall the test statistic from register 00.

STEP FIVE - Evaluating the significance level:

The significance level computed may be considered exact if conducting a one-sided test. After conducting a two-sided test if the significance level computed is less than .1 it may be considered accurate to 4 decimal places. [Ref. 4: pp. 301] Since most alpha levels are at most .1 it is anticipated that virtually all user requirements will be satisfied with this degree of accuracy.

When the data "agree closely" with the hypothesized distribution then the significance level computed in a two-sided test may be considerably larger than the true value; it may even exceed 1. When this happens the user should conclude that the sample data is consistent with the null hypothesis.

C. USER INSTRUCTIONS TO EVALUATE A GIVEN TEST STATISTIC

If a user already knows the value of the test statistic from a K-S test on a continous random variable he may use Continous K-S in the TI-59 to evaluate the significance level. This will provide the user with more precise information than he could get by consulting a table of critical values.

Assume that a K-S test was conducted with sample size n and that the observed value of the test statistic was t. The user should first repartition the calculator by entering "7" and pressing "Op 17". The calculator will display "399.69". Both sides of Card 2 should then be read into the calculator. The significance level (S.L.) of the test may be computed by the following steps:

Enter	Press	Display
n	STO 59	n
t	STO ØØ	t
	E'	S.L.

If the test was one-sided the significance level displayed may be considered exact. If n and t were from a two-sided test the significance level displayed should be doubled. The comments in STEP FIVE of the USER INSTRUCTIONS above apply when using the program to evaluate a known test statistic from a two-sided test.

V. THE K-S TEST: DISCRETE RANDOM VARIABLE

A. THE DISTRIBUTIONS OF TEST STATISTICS

The distributions of the K-S statistics are not independent of the hypothesized distribution, H, when the random variable is discrete. If a test statistic in such a case is evaluated using formula IV. 1 (employed by Continuous K-S) the significance level computed should be viewed as a very conservative upper bound on the true significance level. Conover [Ref. 5] has derived the distribution of the K-S statistics when H is discrete. He has shown that

$$(V.1) \qquad P(D^{+} \geq t) = \sum_{j=\emptyset}^{[n(1-t)]} \binom{n}{j} f_j^{\,n-j} e_j$$

with notation defined as:

1. $[n(1-t)]$ is the largest interger in $n(1-d)$

2. $\binom{n}{j} = n!/j!(n-j)!$

3. $f_j \cdot= P\{X < H^{-1}(1 - t - j/n + eps)\}$

   where X has distribution function H and

$$H^{-1}(p) = \begin{cases} +\infty & , \quad p > 1 \\ \sup\{x;\ H(x) < p\} & , \ \emptyset < p < 1 \\ -\infty & , \quad p < \emptyset \end{cases}$$

4. $e_0 = 1$ and

$$e_k = 1 - \sum_{j=0}^{k-1} \binom{k}{j} f_j^{k-1} e_j , \quad k \geq 1$$

5. The value "eps" is a positive number that approaches, but does not reach, zero. In the notation of the limit arguments from calculus: $eps \to 0^+$.

Similarly,

$$(V.2) \qquad P(D^- \geq t) = \sum_{j=0}^{[n(1-t)]} \binom{n}{j} c_j^{n-j} b_j$$

with additional notation defined as:

1. $c_j = P\{X \geq H(t + j/n)\}$
2. $b_0 = 1$ and

$$b_k = 1 - \sum_{j=0}^{k-1} \binom{k}{j} c_j^{k-1} b_j , \quad k \geq 1 .$$

Finally, for a two-sided test, the distribution of D is closely approximated as follows for small values of t:

(V.3.) $P(D \geq t) = P(D^+ \geq t) + P(D^- \geq t) - P(D^+ \geq t)P(D^- \geq t)/2$

with a maximum error term of $P(D^+ \geq t)P(D^- \geq t)/2$. For significance levels less than or equal to .1 the error will be neglible.

B. USER INSTRUCTIONS FOR "DISCRETE K-S" PROGRAM

The equations defined above require an unacceptable amount of effort to employ by hand computation. The Discrete K-S program implements them on the TI-59 calculator and is listed in Appendix B along with several example problems.

STEP ONE - Checking the Problem Size:

Discrete K-S is designed to test a discrete random variable with m possible outcomes. It may be necessary to aggregate possible outcomes into cells since m must be less than 21. With a sample of size n the program can perform a K-S test if $\{[n(1-t)] + m\} \leq 54$. By convention, $[n(1-t)]$ is defined to be the largest integer in $n(1-t)$. Of course, the value of t, the test statistic, is unknown before the test is conducted but if $(n + m) \leq 54$ then the requirement above is certainly met. If $(n + m)$ is only slightly greater than 54 the data should be entered and processed with Card 1 as explained below. The program will compute the value of the test statistic and indicate if the problem is too large to complete.

STEP TWO - Defining the Hypothesized Distribution:

Each possible value of the random variable being investigated must be classified as falling in one of the m cells. The user must specify the hypothesized distribution, H, in terms of the cells. This may be done in one of two

ways. For each cell i the user can specify either h_i or H_i. For a random observation, X, on the random variable we define h_i and H_i as follows:

$$h_i = P(X \text{ is in cell } i), \qquad i = 1, 2, \ldots, m$$

$$H_i = P(X \text{ is in cell } j;\ j \le i), \qquad i = 1, 2, \ldots, m$$

We call h_i the mass value of cell i and H_i the cumulative mass value of cell i. The number of the n sample observations that fall into cell i is denoted X_i.

STEP THREE - Initializing the Program and Entering the Data:

Read both sides of Card 1 (Discrete K-S) into the calculator and proceed as follows.

Enter	Press	Display
	E (if H defined by h_is)	0
	E′ (if H defined by H_is)	0
m	R/S	m
n	R/S	1
h_1 or H_1	R/S	2
h_2 or H_2	R/S	3
.	.	.
.	.	.
.	.	.
h_{m-1} or H_{m-1}	R/S	m
h_m or H_m	R/S	1 (see Note 1)
X_1	R/S	2
X_2	R/S	3
.	.	.
.	.	.
.	.	.
X_{m-1}	R/S	m
X_m	R/S	0 (see Note 2)

Note 1: If H was entered by mass values h_i , i = 1, 2, . . ., m and ($h_1 + h_2 + . . . + h_m$) is not between .99 and 1.01 then error code "22" will be diplayed after h_m is entered. The user can press "x:t" to display ($h_1 + h_2 + . . . + h_m$). Ideally, the mass values will always sum to 1 but the user may be using rounded values which do not. If the user wishes to proceed with the K-S test after error code "22" is displayed he should press "A′". This will cause the mass values to be normalized so they will sum to 1. Normally, this should only be done if a more accurate statement of mass values is unavailable.

Note 2: If ($X_1 + X_2 + . . . + X_m$) $\neq$ n then error code "33" will be displayed. The test cannot proceed.

STEP FOUR - Computing the Test Statistic:

After completing the Initialization and Data Entry sequence the test statistic, t, should be computed.

If H_o: $F(x) = H(x)$ then press A.

If H_o: $F(x) \geq H(x)$ then press B.

If H_o: $F(x) \leq H(x)$ then press C.

After processing the data and computing the appropriate test statistic, t, the calculator will either display t or "44". The latter indicates that the problem is too large for Discrete K-S. The user can recall t from register 00. If t is displayed (always a number between 0 and 1) then the user can proceed with the test.

STEP FIVE - Computing the Significance Level:

After the test statistic is displayed the program has automatically repartitioned the calculator to allow Card 2 to be read into the calculator. The user is cautioned not to press "CP" or "RST" between using Card 1 and Card 2. This is important because certain flags are set in the calculator during the previous steps.

To compute the significance level of the test simply read Card 2, both sides, into the calculator and press "A". If a one-sided test is conducted, then the significance level displayed may be considered exact. If a two-sided test is conducted then the calculator displays an approximate significance level. The following steps may be used to judge the quality of the approximation:

Press	Display
B	upper bound on S.L.
B, $x \rightleftarrows t$	lower bound on S.L.
C	maximum error in point estimate of S.L.
D	recall point estimate of S.L.

After the test is completed the calculator can be restored to normal partitioning by pressing "E".

The user should be aware that the time required to compute a significance level can be considerable. The following table may be used to estimate computing time.

Problem Size:		Time to compute significance level:	
n	m	one-sided test	two-sided test
10	5	3 min, 30 sec	7 min
20	10	45 min	90 min
40	12	2 hours	4 hours

If the user needs the capability to handle larger problems and/or process data quickly a FORTRAN program has been developed by Allen [Ref. 9] for this purpose.

APPENDIX A

CONTINUOUS K-S: ENTRY PROCEDURE, LISTING, AND EXAMPLES

A. ENTRY PROCEDURE

The program is organized in two distinct parts, Card 1 and Card 2. Card 1 should be recorded on a magnetic card using banks 1 and 2 [Ref. 8: pp. VII-2] with the calculator in standard partitioning [Ref. 8: pp. V-22]. Card 2 should also be recorded on banks 1 and 2 but with partitioning 399.69.

It should be noted in the program listing below that Cards 1 and 2 both use the "Dsz" command followed by register numbers greater than 9. The user's manual [Ref. 8: pp. V-63] implies that this will not work but, in fact, it does. The user has to use a special entry technique to get the program steps into the calculator correctly, however. One very convenient way to accomplish this is illustrated easily by an example.

Assume that the steps at one point in the program are "Dsz, 58". The user can first enter "STO, 58", then backstep twice until "42" is in the display (for "STO"). He should then overwrite the "STO" command by pressing "Dsz". This will leave "58" in the display. He should then advance the program pointer one step (with "SST") and continue entering the rest of the program.

B. CONTINUOUS K-S PROGRAM LISTING

CARD 1

```
000  76  LBL
001  96  WRT
002  22  INV
003  86  STF
004  03   03
005  53   (
006  53   (
007  43  RCL
008  68   68
009  23  LNX
010  55   ÷
011  02   2
012  23  LNX
013  54   )
014  59  INT
015  32  X:T
016  02   2
017  45  YX
018  32  X:T
019  54   )
020  59  INT
021  42  STD
022  67   67
023  29  CP
024  67   EQ
025  38  SIN
026  87  IFF
027  03   03
028  00   00
029  83   83
030  86  STF
031  03   03
032  25  CLR
033  42  STD
034  65   65
035  42  STD
036  66   66
037  43  RCL
038  67   67
039  44  SUM
040  66   66
041  53   (
042  94  +/-
043  85   +
044  01   1
045  85   +
046  43  RCL
047  68   68
048  54   )
049  42  STD
050  69   69
051  29  CP
052  53   (
053  73  RC*
054  66   66
055  75   -
056  73  RC*
057  65   65
058  54   )
059  77   GE
060  00   00
061  71   71
062  63  EX*
063  66   66
064  63  EX*
065  65   65
066  63  EX*
067  66   66
068  22  INV
069  86  STF
070  03   03
071  01   1
072  44  SUM
073  65   65
074  44  SUM
075  66   66
076  97  DSZ
077  69   69
078  00   00
079  51   51
080  61  GTD
081  00   00
082  26   26
083  22  INV
084  86  STF
085  03   03
086  53   (
087  43  RCL
088  67   67
```

089	55	÷
090	02	2
091	54	)
092	61	GTO
093	00	00
094	20	20
095	25	CLR
096	42	STO
097	68	68
098	92	RTN
099	76	LBL
100	15	E
101	25	CLR
102	07	7
103	69	OP
104	17	17
105	86	STF
106	08	08
107	47	CMS
108	92	RTN
109	76	LBL
110	11	A
111	99	PRT
112	72	ST*
113	68	68
114	32	X:T
115	01	1
116	44	SUM
117	68	68
118	44	SUM
119	59	59
120	43	RCL
121	68	68
122	92	RTN
123	76	LBL
124	13	C
125	71	SBR
126	02	02
127	85	85
128	01	1
129	22	INV
130	44	SUM
131	68	68
132	71	SBR
133	96	WRT
134	98	ADV
135	98	ADV
136	71	SBR
137	02	02
138	60	60
139	98	ADV
140	01	1
141	22	INV
142	90	LST
143	92	RTN
144	76	LBL
145	38	SIN
146	43	RCL
147	59	59
148	42	STO
149	58	58
150	75	-
151	01	1
152	54	)
153	42	STO
154	57	57
155	73	RC*
156	57	57
157	72	ST*
158	58	58
159	01	1
160	94	+/-
161	44	SUM
162	57	57
163	97	DSZ
164	58	58
165	01	01
166	55	55
167	92	RTN
168	76	LBL
169	16	A'
170	43	RCL
171	59	59
172	42	STO
173	58	58
174	73	RC*
175	58	58
176	23	LNX
177	72	ST*
178	58	58
179	97	DSZ
180	58	58

CONTINUOUS K-S, CARD 1 (con't)

```
181  01  01
182  74  74
183  18  C'
184  92  RTN
185  76  LBL
186  17  B'
187  43  RCL
188  59  59
189  42  STO
190  58  58
191  73  RC*
192  58  58
193  71  SBR
194  10  E'
195  72  ST*
196  58  58
197  97  DSZ
198  58  58
199  01  01
200  91  91
201  18  C'
202  92  RTN
203  76  LBL
204  18  C'
205  98  ADV
206  98  ADV
207  03  3
208  07  7
209  03  3
210  05  5
211  01  1
212  03  3
213  69  OP
214  01  01
215  03  3
216  01  1
217  03  3
218  06  6
219  02  2
220  01  1
221  03  3
222  02  2
223  03  3
224  05  5
225  69  OP
226  02  02
227  03  3
228  00  0
229  01  1
230  07  7
231  01  1
232  06  6
233  00  0
234  00  0
235  01  1
236  06  6
237  69  OP
238  03  03
239  01  1
240  03  3
241  03  3
242  07  7
243  01  1
244  03  3
245  00  0
246  00  0
247  00  0
248  00  0
249  69  OP
250  04  04
251  69  OP
252  05  05
253  69  OP
254  00  00
255  98  ADV
256  01  1
257  22  INV
258  90  LST
259  92  RTN
260  02  2
261  04  4
262  03  3
263  01  1
264  03  3
265  03  3
266  04  4
267  01  1
268  03  3
269  07  7
270  69  OP
271  02  02
272  01  1
```

CONTINUOUS K-S, CARD 1 (con't)

273	06	6
274	01	1
275	03	3
276	03	3
277	07	7
278	01	1
279	03	3
280	69	OP
281	03	03
282	69	OP
283	05	05
284	92	RTN
285	43	RCL
286	59	59
287	75	-
288	01	1
289	54	)
290	42	STO
291	58	58
292	73	RC*
293	58	58
294	32	X:T
295	01	1
296	22	INV
297	44	SUM
298	58	58
299	73	RC*
300	58	58
301	32	X:T
302	77	GE
303	03	03
304	06	06
305	92	RTN
306	01	1
307	44	SUM
308	58	58
309	97	DSZ
310	58	58
311	02	02
312	92	92
313	71	SBR
314	38	SIN
315	98	ADV
316	98	ADV
317	71	SBR
318	02	02
319	60	60
320	98	ADV
321	01	1
322	22	INV
323	90	LST
324	91	R/S

CONTINUOUS K-S, CARD 2

```
000  76 LBL
001  19 D'
002  77  GE
003  00  00
004  06  06
005  32 X:T
006  92 RTN
007  76 LBL
008  12  B
009  86 STF
010  04  04
011  61 GTO
012  11  A
013  76 LBL
014  13  C
015  86 STF
016  05  05
017  76 LBL
018  11  A
019  25 CLR
020  91 R/S
021  76 LBL
022  16 A'
023  42 STO
024  63  63
025  91 R/S
026  42 STO
027  62  62
028  61 GTO
029  00  00
030  55  55
031  76 LBL
032  17 B'
033  86 STF
034  01  01
035  42 STO
036  63  63
037  01  1
038  42 STO
039  62  62
040  91 R/S
041  42 STO
042  62  62
043  61 GTO
044  00  00
045  55  55
046  76 LBL
047  18 C'
048  86 STF
049  02  02
050  42 STO
051  63  63
052  91 R/S
053  42 STO
054  62  62
055  00  0
056  42 STO
057  68  68
058  42 STO
059  67  67
060  42 STO
061  65  65
062  43 RCL
063  59  59
064  42 STO
065  66  66
066  85  +
067  01  1
068  42 STO
069  64  64
070  54  )
071  42 STO
072  00  00
073  89  π
074  94 +/-
075  72 ST*
076  00  00
077  73 RC*
078  64  64
079  32 X:T
080  01  1
081  44 SUM
082  64  64
083  73 RC*
084  64  64
085  67  EQ
086  01  01
087  46  46
088  01  1
```

CONTINUOUS K-S, CARD 2 (con't)

089	22	INV
090	44	SUM
091	64	64
092	32	X:T
093	71	SBR
094	01	01
095	83	83
096	42	STO
097	00	00
098	43	RCL
099	64	64
100	55	÷
101	43	RCL
102	59	59
103	54	)
104	75	-
105	43	RCL
106	00	00
107	54	)
108	32	X:T
109	43	RCL
110	68	68
111	19	D'
112	42	STO
113	68	68
114	43	RCL
115	64	64
116	75	-
117	01	1
118	75	-
119	43	RCL
120	65	65
121	54	)
122	55	÷
123	43	RCL
124	59	59
125	54	)
126	94	+/-
127	85	+
128	43	RCL
129	00	00
130	54	)
131	32	X:T
132	43	RCL
133	67	67
134	19	D'
135	42	STO
136	67	67
137	00	0
138	42	STO
139	65	65
140	01	1
141	44	SUM
142	64	64
143	61	GTO
144	01	01
145	49	49
146	01	1
147	44	SUM
148	65	65
149	97	DSZ
150	66	66
151	00	00
152	77	77
153	87	IFF
154	04	04
155	01	01
156	73	73
157	87	IFF
158	05	05
159	01	01
160	78	78
161	43	RCL
162	67	67
163	32	X:T
164	43	RCL
165	68	68
166	19	D'
167	10	E'
168	65	×
169	02	2
170	54	)
171	99	PRT
172	91	R/S
173	43	RCL
174	68	68
175	10	E'
176	99	PRT
177	91	R/S
178	43	RCL
179	67	67
180	10	E'

```
181  99  PRT
182  91  R/S
183  87  IFF
184  01   01
185  02   02
186  03   03
187  87  IFF
188  02   02
189  02   02
190  19   19
191  75   -
192  43  RCL
193  63   63
194  54   )
195  55   ÷
196  43  RCL
197  62   62
198  54   )
199  36  PGM
200  19   19
201  12   B
202  92  RTN
203  65   ×
204  43  RCL
205  63   63
206  54   )
207  45  Yˣ
208  43  RCL
209  62   62
210  54   )
211  94  +/-
212  22  INV
213  23  LNX
214  94  +/-
215  85   +
216  01   1
217  54   )
218  92  RTN
219  32  X:T
220  43  RCL
221  63   63
222  77   GE
223  02   02
224  48   48
225  43  RCL
226  62   62
227  22  INV
228  77   GE
229  02   02
230  50   50
231  32  X:T
232  75   -
233  43  RCL
234  63   63
235  54   )
236  32  X:T
237  43  RCL
238  62   62
239  75   -
240  43  RCL
241  63   63
242  54   )
243  35  1/X
244  65   ×
245  32  X:T
246  54   )
247  92  RTN
248  00   0
249  92  RTN
250  01   1
251  92  RTN
252  76  LBL
253  10  E'
254  42  STO
255  00   00
256  01   1
257  75   -
258  43  RCL
259  00   00
260  54   )
261  42  STO
262  66   66
263  65   ×
264  43  RCL
265  59   59
266  54   )
267  59  INT
268  85   +
269  01   1
270  54   )
271  42  STO
272  68   68
```

CONTINUOUS K-S, CARD 2 (con't)

```
273  43 RCL
274  59  59
275  42 STO
276  67  67
277  35 1/X
278  42 STO
279  58  58
280  43 RCL
281  00  00
282  42 STO
283  57  57
284  00  0
285  42 STO
286  65  65
287  42 STO
288  61  61
289  01  1
290  94 +/-
291  42 STO
292  56  56
293  71 SBR
294  03  03
295  46  46
296  42 STO
297  60  60
298  43 RCL
299  66  66
300  45 YX
301  43 RCL
302  67  67
303  54  )
304  49 PRD
305  60  60
306  43 RCL
307  57  57
308  45 YX
309  43 RCL
310  56  56
311  54  )
312  49 PRD
313  60  60
314  43 RCL
315  60  60
316  44 SUM
317  61  61
318  01  1
319  44 SUM
320  65  65
321  44 SUM
322  56  56
323  22 INV
324  44 SUM
325  67  67
326  43 RCL
327  58  58
328  22 INV
329  44 SUM
330  66  66
331  44 SUM
332  57  57
333  97 DSZ
334  68  68
335  02  02
336  93  93
337  43 RCL
338  61  61
339  65  X
340  43 RCL
341  00  00
342  54  )
343  98 ADV
344  98 ADV
345  92 RTN
346  43 RCL
347  65  65
348  42 STO
349  63  63
350  32 X:T
351  00  0
352  22 INV
353  67  EQ
354  03  03
355  58  58
356  01  1
357  92 RTN
358  01  1
359  42 STO
```

CONTINUOUS K-S, CARD 2 (con't)

360	62	62
361	43	RCL
362	63	63
363	94	+/-
364	85	+
365	43	RCL
366	59	59
367	42	STO
368	64	64
369	54	)
370	32	X:T
371	43	RCL
372	63	63
373	22	INV
374	77	GE
375	03	03
376	80	80
377	32	X:T
378	42	STO
379	63	63
380	43	RCL
381	64	64
382	55	÷
383	43	RCL
384	63	63
385	54	)
386	49	PRD
387	62	62
388	01	1
389	22	INV
390	44	SUM
391	64	64
392	97	DSZ
393	63	63
394	03	03
395	80	80
396	43	RCL
397	62	62
398	92	RTN

C. CONTINUOUS K-S EXAMPLE PROBLEMS

1. Example 1

a. Problem:

H_0 : $F(x) = H(x)$ for all x, where H = Uniform (0,5), i.e.,

$$H(x) = \begin{cases} 1 & , \quad x > 5 \\ x/5 & , \quad 0 \leq x \leq 5 \\ 0 & , \quad x < 0 \end{cases}$$

H_1 : $F(x) \neq H(x)$ for some x

Sample size = n = 10

Sample data: 0.8, 4.0, 0.2, 2.6, 3.8, 0.6, 1.0, 4.8, 1.2, 1.4

b. Solution:

Read both sides of Card 1 into the calculator, then:

ENTER	PRESS	DISPLAY	PRINTOUT	COMMENTS
	E	399.69		new partitioning
0.8	A	1	0.8	sample data
4.0	A	2	4.0	
.	.	.	.	
.	.	.	.	
.	.	.	.	
.	.	.	.	
1.4	A	10	1.4	
	C	1	INPUT DATA	[Time: 1 min, 20 sec]
			0.2 01	sorted sample data
			0.6 02	
			.	
			.	
			.	
			4.8 10	
			0. 11	ignore
			0. 12	
			.	
			.	
			.	

(con't)

After completing the previous steps using Card 1 read both sides of Card 2 into the calculator. Do not press "CP" or "RST". Complete the test as follows:

ENTER	PRESS	DISPLAY	PRINTOUT	COMMENTS
	A	Ø		two-tailed test
Ø	C	Ø		H = Uniform (Ø,5)
5	R/S	.2Ø73748895	.2Ø73748895	significance level [Time: 1 min, 33 sec]

Since the test conducted was a two-tailed test the significance level computed is approximate. The user is assured that the true significance level is no greater than the value computed. The user may recall the test statistic t = .32 from register ØØ.

2. Example 2

a. Problem:

H_0 : $F(x) \leq H(x)$ for all x, where H = Normal $(3, 2^2)$

H_1 : $F(x) > H(x)$ for some x

Sample size = n = 5

Sample data: 1.462, -0.311, 0.555, 5.711, -0.078

b. Solution:

Read both sides of card 1 into the calculator, then:

ENTER	PRESS	DISPLAY	PRINTOUT	COMMENTS
	E	399.69		new partitioning
1.462	A	1	1.462	sample data
-0.311	A	2	-0.311	
0.555	A	3	0.555	
5.711	A	4	5.711	
-0.078	A	5	-0.078	
	C	1	INPUT DATA	[Time: 30 sec]
			-0.311 01	Sorted Sample Data
			-0.078 02	
			0.555 03	
			1.462 04	
			5.711 05	
			0. 06	ignore
			0. 07	
			.	
			.	
			.	

Read Card 2, both sides, into the calculator, then:

ENTER	PRESS	DISPLAY	PRINTOUT	COMMENTS
	B	0	.	one-tailed test
3	A'	3		H = Normal $(3, 2^2)$
2	R/S	.0201689866	.0201689866	significance level [Time: 53 sec]

The user may recall the test statistic t = .5790534154 from register 00.

3. Example 3

a. Problem:

H_0: $F(x) = H(x)$ for all x, where

$$H(x) = \begin{cases} 0 , & x < 0 \\ x^2/4 , & 0 \le x \le 2 \\ 1 , & x > 2 \end{cases}$$

H_1: $F(x) \neq H(x)$ for some x

Sample size n = 10

Sample data: 1.8, 1.0, 1.3, 0.5, 1.98, 0.95, 1.91, 0.75, 1.85, 1.6

b. Discussion:

Continuous K-S does not have a subroutine to evaluate the hypothesized CDF H for this problem. This is an ideal situation to invoke the probability integral transformation. Under the null hypothesis X has CDF H, thus, H(X) is distributed uniform (0,1) under the null hypothesis. If we transorm each sample data value with H, defined above, then we can proceed with the K-S test using a uniform (0,1) distribution in place of the orginal hypothesized distribution.

c. Solution:

Read both sides of Card 1 into the calculator, then:

ENTER	PRESS	DISPLAY	PRINTOUT	COMMENT
	E	399.69		new partitioning
1.8	A	1	1.8	
1.0	A	2	1.0	
.	.	.	.	sample data
.	.	.	.	
.	.	.	.	
1.6	A	10	1.6	
	C	1	INPUT DATA	[Time: 1 min, 20 sec]
			0.5 01	
			0.75 02	
			.	sorted sample data
			.	
			.	
			1.98 10	
			0. 11	
			0. 12	
			.	ignore
			.	
			.	
	GTO 325	1		
	LRN	325 00		
	LBL	326 00		
	E′	327 00		
	x^2	328 00		Transformation Subroutine H
	÷	329 00		
	4	330 00		
	)	331 00		
	INV SBR	332 00		
	LRN	1		
2	E′	1		Subroutine test
	B′	TRANSFORMED DATA		
		0.0625 01		
		0.140625 02		
		0.225625 03		
		0.25 04		
		0.4225 05		Transformed data
		0.64 06		
		0.81 07		
		0.855625 08		
		0.912025 09		
		0.9801 10		
		0. 11		
		0. 12		ignore
		.		
		.		

Read both sides of Card 2 into the calculator, then:

ENTER	PRESS	DISPLAY	PRINTOUT	COMMENT
	A	Ø		two-sided test
Ø	C´	Ø		uniform (Ø,1)
1	R/S	.7275Ø48241	.7275Ø48421	upper bound on significance level [Time: 2 min]

If the significance level computed had been .1 or less we could consider it exact. With as value as large as that computed, however, we should consider it an upper bound on the true sgnificance level and conclude that the sample data and the null hypothesis are in close agreement. The user may recall the test statistic t = .21 from register ØØ.

4. Example 4

a. Problem:

H_0 : $F(x) \geq H(x)$ for all x, where H = lognormal $(3, 5^2)$

H_1 : $F(x) < H(x)$ for some x

Sample size = n = 7

Sample data: 99.31, 22.47, .0608, 3294.5, 4.95, 14.88, 29.96

b. Discussion:

This problem demonstrates the use of the natural log transformation. It is appropriate when the hypothesized distribution is lognormal. Recall that X is distributed lognormal (μ, σ^2) if and only if ln(X) is distributed normal (μ, σ^2). We rely on the natural log transformation subroutine incorporated into Card 1 of Continuous K-S to transform our sample data. We then test the transformed data against a normal $(3, 5^2)$ CDF using Card 2.

c. Solution:

Read both sides of Card 1 into the calculator, then:

ENTER	PRESS	DISPLAY	PRINTOUT	COMMENT
	E	399.69		new partitioning
99.31	A	1	99.31	sample data
22.47	A	2	22.47	
0.0608	A	3	0.0608	
3294.5	A	4	3294.5	
4.95	A	5	4.95	
14.88	A	6	14.88	
29.96	A	7	29.96	

ENTER	PRESS	DISPLAY	PRINTOUT	COMMENT
	C	1	INPUT DATA	[Time: 46 sec]
			0.0608 01	sorted sample data
			4.95 02	
			14.88 03	
			22.47 04	
			29.96 05	
			99.31 06	
			3294.5 07	
			0. 08	ignore
			0. 09	
			.	
			.	
			.	
	A′	1	TRANSFORMED DATA	
			-2.80016549 01	Transformed data
			1.599387577 02	
			2.700018029 03	
			3.112181086 04	
			3.399863159 05	
			4.598246271 06	
			8.10000969 07	
			0. 08	ignore
			0. 09	
			.	
			.	
			.	

Read both sides of Card 2 into the calculator, then:

ENTER	PRESS	DISPLAY	PRINTOUT	COMMENT
	$\bar{C}$	0		One-sided test
3	A′	3		normal (3, 5^2)
5	R/S	.3672246357	.3672246357	significance level [Time 1 min, 25 sec]

The user may recall test statistic t = .2468346856 from register 00.

APPENDIX B

DISCRETE K-S: ENTRY PROCEDURE, LISTING, AND EXAMPLES

A. ENTRY PROCEDURE

The program is organized in two distinct parts, Card 1 and Card 2. Card 1 should be recorded on a magnetic card using banks 1 and 2 [Ref. 8: pp. VII-2] with the calculator in standard partitioning [Ref. 8: pp. V-22]. Card 2 should also be recorded in banks 1 and 2 but with partitioning 399.69.

It should be noted in the program lising below that Cards 1 and 2 both use the "Dsz" command followed by register numbers greater than 9. The user's manual [Ref. 8: pp. V-63] implies that this will not work but, in fact, it does. The user has to use a special entry technique to get the program steps into the calculator correctly, however. One very convenient way to accomplish this is illustrated easily by an example.

Assume the steps at one point in the program are "Dsz, 58". The user can first enter "STO, 58", then backstep twice until "42" is in the display (for "STO"). He should then overwrite the "STO" command by pressing "Dsz". This will leave "58" in the display. He should advance the program pointer one step (with "SST") and continue entering the rest of the program.

Discrete K-S uses the "HIR" command, explained in detail in Appendix D. It is represented by machine code "82" and thus must be entered with a special procedure. A convenient way to enter "HIR" is to precede it with "STO", enter "82", and backstep twice, leaving "42" (for "STO") in the display. Delete the "STO" command with "del", advance the program pointer one step, and continue entering the program. When "HIR" is followed by another two-digit merged code, e.g., "HIR, 16", the entry procedure should be used twice, the first time to enter "82" and the second time to enter "16".

B. DISCRETE K-S PROGRAM LISTING

CARD 1

```
000  76 LBL
001  15  E
002  25 CLR
003  47 CMS
004  86 STF
005  00  00
006  61 GTO
007  00  00
008  16  16
009  76 LBL
010  10 E'
011  22 INV
012  86 STF
013  00  00
014  25 CLR
015  47 CMS
016  22 INV
017  86 STF
018  04  04
019  91 R/S
020  42 STO
021  41  41
022  85  +
023  01  1
024  54  )
025  32 X:T
026  43 RCL
027  41  41
028  91 R/S
029  42 STO
030  42  42
031  01  1
032  42 STO
033  00  00
034  91 R/S
035  72 ST*
036  00  00
037  22 INV
038  87 IFF
039  00  00
040  00  00
041  44  44
042  44 SUM
043  45  45
044  69 OP
045  20  20
046  43 RCL
047  00  00
048  22 INV
049  77  GE
050  00  00
051  34  34
052  22 INV
053  87 IFF
054  00  00
055  16 A'
056  01  1
057  93  .
058  00  0
059  01  1
060  32 X:T
061  43 RCL
062  45  45
063  77  GE
064  03  03
065  87  87
066  32 X:T
067  93  .
068  09  9
069  09  9
070  77  GE
071  03  03
072  87  87
073  76 LBL
074  16 A'
075  43 RCL
076  41  41
077  85  +
078  01  1
079  54  )
080  32 X:T
081  02  2
082  01  1
083  42 STO
084  00  00
085  01  1
086  42 STO
087  44  44
088  91 R/S
```

```
089  72  ST÷
090  00   00
091  44  SUM
092  43   43
093  69  OP
094  20   20
095  01   1
096  44  SUM
097  44   44
098  43  RCL
099  44   44
100  22  INV
101  77   GE
102  00   00
103  88   88
104  43  RCL
105  42   42
106  32  X:T
107  43  RCL
108  43   43
109  22  INV
110  67   EQ
111  03   03
112  91   91
113  25  CLR
114  91  R/S
115  76  LBL
116  11   A
117  86  STF
118  01   01
119  22  INV
120  86  STF
121  02   02
122  61  GTO
123  01   01
124  42   42
125  76  LBL
126  12   B
127  22  INV
128  86  STF
129  02   02
130  22  INV
131  86  STF
132  01   01
133  61  GTO
134  01   01
135  42   42
136  76  LBL
137  13   C
138  86  STF
139  01   01
140  86  STF
141  02   02
142  87  IFF
143  04   04
144  03   03
145  01   01
146  86  STF
147  04   04
148  22  INV
149  87  IFF
150  00   00
151  01   01
152  70   70
153  43  RCL
154  45   45
155  35  1/X
156  42  STO
157  45   45
158  43  RCL
159  41   41
160  42  STO
161  44   44
162  43  RCL
163  45   45
164  64  PD*
165  44   44
166  97  DSZ
167  44   44
168  01   01
169  64   64
170  43  RCL
171  42   42
172  35  1/X
173  42  STO
174  43   43
175  02   2
176  01   1
177  42  STO
178  00   00
179  43  RCL
180  41   41
```

DISCRETE K-S, CARD 1 (con't)

```
181  42  STO
182  44   44
183  43  RCL
184  43   43
185  64  PD*
186  00   00
187  01   1
188  44  SUM
189  00   00
190  97  DSZ
191  44   44
192  01   01
193  83   83
194  43  RCL
195  41   41
196  75   -
197  01   1
198  54   )
199  42  STO
200  44   44
201  02   2
202  01   1
203  42  STO
204  00   00
205  01   1
206  42  STO
207  43   43
208  73  RC*
209  00   00
210  69  OP
211  20   20
212  74  SM*
213  00   00
214  22  INV
215  87  IFF
216  00   00
217  02   02
218  28   28
219  73  RC*
220  43   43
221  32  X:T
222  01   1
223  44  SUM
224  43   43
225  32  X:T
226  74  SM*
227  43   43
228  97  DSZ
229  44   44
230  02   02
231  08   08
232  01   1
233  42  STO
234  00   00
235  42  STO
236  50   50
237  42  STO
238  51   51
239  02   2
240  01   1
241  42  STO
242  43   43
243  43  RCL
244  41   41
245  42  STO
246  44   44
247  00   0
248  42  STO
249  47   47
250  42  STO
251  48   48
252  73  RC*
253  00   00
254  75   -
255  73  RC*
256  43   43
257  54   )
258  42  STO
259  45   45
260  94  +/-
261  32  X:T
262  43  RCL
263  48   48
264  77   GE
265  02   02
266  77   77
267  32  X:T
268  42  STO
269  48   48
270  43  RCL
271  00   00
272  42  STO
```

DISCRETE K-S, CARD 1 (con´t)

273	50	50
274	61	GTO
275	02	02
276	92	92
277	43	RCL
278	45	45
279	32	X:T
280	43	RCL
281	47	47
282	77	GE
283	02	02
284	92	92
285	32	X:T
286	42	STO
287	47	47
288	43	RCL
289	00	00
290	42	STO
291	51	51
292	01	1
293	44	SUM
294	00	00
295	44	SUM
296	43	43
297	97	DSZ
298	44	44
299	02	02
300	52	52
301	87	IFF
302	01	01
303	03	03
304	12	12
305	43	RCL
306	47	47
307	82	HIR
308	04	04
309	61	GTO
310	03	03
311	31	31
312	87	IFF
313	02	02
314	03	03
315	27	27
316	43	RCL
317	47	47
318	32	X:T
319	43	RCL
320	48	48
321	77	GE
322	03	03
323	27	27
324	61	GTO
325	03	03
326	05	05
327	43	RCL
328	48	48
329	82	HIR
330	04	04
331	07	7
332	69	OP
333	17	17
334	01	1
335	75	-
336	82	HIR
337	14	14
338	95	=
339	65	×
340	43	RCL
341	42	42
342	95	=
343	85	+
344	02	2
345	95	=
346	59	INT
347	42	STO
348	59	59
349	43	RCL
350	42	42
351	82	HIR
352	02	02
353	43	RCL
354	41	41
355	82	HIR
356	03	03
357	05	5
358	05	5
359	75	-

DISCRETE K-S, CARD 1 (con´t)

360	82	HIR
361	13	13
362	95	=
363	32	X:T
364	43	RCL
365	59	59
366	77	GE
367	03	03
368	84	84
369	93	.
370	00	0
371	00	0
372	00	0
373	00	0
374	00	0
375	00	0
376	00	0
377	00	0
378	01	1
379	42	STD
380	56	56
381	82	HIR
382	14	14
383	91	R/S
384	04	4
385	04	4
386	91	R/S
387	32	X:T
388	02	2
389	02	2
390	91	R/S
391	32	X:T
392	03	3
393	03	3
394	91	R/S

DISCRETE K-S, CARD 2

```
000  76 LBL
001  12  B
002  82 HIR
003  13  13
004  75  -
005  82 HIR
006  12  12
007  95  =
008  32 X:T
009  82 HIR
010  13  13
011  85  +
012  82 HIR
013  12  12
014  95  =
015  91 R/S
016  76 LBL
017  13  C
018  82 HIR
019  12  12
020  91 R/S
021  76 LBL
022  14  D
023  82 HIR
024  13  13
025  91 R/S
026  76 LBL
027  11  A
028  22 INV
029  87 IFF
030  01  01
031  00  00
032  81  81
033  22 INV
034  87 IFF
035  02  02
036  00  00
037  43  43
038  71 SBR
039  88 DMS
040  82 HIR
041  06  06
042  91 R/S
043  71 SBR
044  88 DMS
045  82 HIR
046  06  06
047  22 INV
048  86 STF
049  01  01
050  71 SBR
051  88 DMS
052  82 HIR
053  05  05
054  85  +
055  82 HIR
056  16  16
057  95  =
058  42 STO
059  00  00
060  82 HIR
061  15  15
062  65  X
063  82 HIR
064  16  16
065  55  ÷
066  02  2
067  95  =
068  82 HIR
069  02  02
070  32 X:T
071  82 HIR
072  12  12
073  94 +/-
074  85  +
075  43 RCL
076  00  00
077  95  =
078  82 HIR
079  03  03
080  91 R/S
081  71 SBR
082  88 DMS
083  82 HIR
084  05  05
085  91 R/S
086  76 LBL
087  88 DMS
088  82 HIR
```

DISCRETE K-S, CARD 2 (con't)

089	13	13
090	85	÷
091	01	1
092	95	=
093	42	STO
094	62	62
095	42	STO
096	58	58
097	01	1
098	72	ST*
099	62	62
100	02	2
101	42	STO
102	00	00
103	01	1
104	82	HIR
105	07	07
106	00	0
107	42	STO
108	60	60
109	01	1
110	42	STO
111	69	69
112	42	STO
113	64	64
114	43	RCL
115	59	59
116	42	STO
117	68	68
118	43	RCL
119	69	69
120	71	SBR
121	03	03
122	05	05
123	32	X:T
124	00	0
125	67	EQ
126	01	01
127	55	55
128	32	X:T
129	45	Y^{X}
130	43	RCL
131	64	64
132	95	=
133	42	STO
134	57	57
135	43	RCL
136	69	69
137	75	-
138	01	1
139	95	=
140	82	HIR
141	08	08
142	71	SBR
143	02	02
144	68	68
145	49	PRD
146	57	57
147	73	RC*
148	62	62
149	49	PRD
150	57	57
151	43	RCL
152	57	57
153	44	SUM
154	60	60
155	01	1
156	44	SUM
157	69	69
158	44	SUM
159	62	62
160	94	+/-
161	44	SUM
162	64	64
163	43	RCL
164	00	00
165	32	X:T
166	43	RCL
167	69	69
168	22	INV
169	77	GE
170	01	01
171	18	18
172	01	1
173	75	-
174	43	RCL
175	60	60
176	95	=
177	72	ST*
178	62	62
179	01	1
180	42	STO

DISCRETE K-S, CARD 2 (con't)

181	69	69
182	44	SUM
183	00	00
184	82	HIR
185	37	37
186	82	HIR
187	17	17
188	42	STO
189	64	64
190	43	RCL
191	68	68
192	32	X:T
193	43	RCL
194	00	00
195	77	GE
196	02	02
197	08	08
198	00	0
199	42	STO
200	60	60
201	43	RCL
202	58	58
203	42	STO
204	62	62
205	61	GTO
206	01	01
207	18	18
208	01	1
209	94	+/-
210	44	SUM
211	68	68
212	43	RCL
213	58	58
214	42	STO
215	62	62
216	00	0
217	82	HIR
218	08	08
219	42	STO
220	61	61
221	82	HIR
222	12	12
223	82	HIR
224	07	07
225	42	STO
226	67	67
227	71	SBR
228	02	02
229	68	68
230	42	STO
231	60	60
232	73	RC*
233	62	62
234	49	PRD
235	60	60
236	43	RCL
237	69	69
238	71	SBR
239	03	03
240	05	05
241	45	YX
242	43	RCL
243	67	67
244	95	=
245	49	PRD
246	60	60
247	43	RCL
248	60	60
249	44	SUM
250	61	61
251	01	1
252	44	SUM
253	62	62
254	44	SUM
255	69	69
256	82	HIR
257	38	38
258	94	+/-
259	44	SUM
260	67	67
261	97	DSZ
262	68	68
263	02	02
264	27	27
265	43	RCL
266	61	61
267	92	RTN
268	00	0
269	32	X:T
270	82	HIR
271	18	18
272	67	EQ

DISCRETE K-S, CARD 2 (con't)

273	03	03
274	03	03
275	42	STD
276	66	66
277	82	HIR
278	17	17
279	42	STD
280	65	65
281	01	1
282	42	STD
283	63	63
284	43	RCL
285	65	65
286	55	÷
287	43	RCL
288	66	66
289	95	=
290	49	PRD
291	63	63
292	01	1
293	94	+/-
294	44	SUM
295	65	65
296	97	DSZ
297	66	66
298	02	02
299	84	84
300	43	RCL
301	63	63
302	92	RTN
303	01	1
304	92	RTN
305	22	INV
306	87	IFF
307	01	01
308	03	03
309	64	64
310	94	+/-
311	85	+
312	82	HIR
313	12	12
314	85	+
315	01	1
316	95	=
317	55	÷
318	82	HIR
319	12	12
320	95	=
321	75	-
322	82	HIR
323	14	14
324	95	=
325	85	+
326	43	RCL
327	56	56
328	95	=
329	32	X:T
330	43	RCL
331	01	01
332	77	GE
333	03	03
334	91	91
335	82	HIR
336	13	13
337	42	STD
338	63	63
339	73	RC*
340	63	63
341	22	INV
342	77	GE
343	03	03
344	49	49
345	97	DSZ
346	63	63
347	03	03
348	39	39
349	22	INV
350	87	IFF
351	01	01
352	03	03
353	55	55
354	92	RTN
355	01	1
356	44	SUM
357	63	63
358	01	1
359	75	-

360	73	RC*
361	63	63
362	95	=
363	92	RTN
364	75	-
365	01	1
366	95	=
367	55	÷
368	82	HIR
369	12	12
370	95	=
371	85	+
372	82	HIR
373	14	14
374	75	-
375	43	RCL
376	56	56
377	95	=
378	32	X:T
379	82	HIR
380	13	13
381	75	-
382	01	1
383	95	=
384	42	STO
385	63	63
386	73	RC*
387	63	63
388	77	GE
389	03	03
390	35	35
391	00	0
392	92	RTN
393	76	LBL
394	15	E
395	06	6
396	69	OP
397	17	17
398	91	R/S

C. DISCRETE K-S EXAMPLE PROBLEMS

1. Example 1

a. Problem:

H_o : $F(x) = H(x)$ for all x, where H is defined below

H_1 : $F(x) \neq H(x)$ for some x

Number of cells = m = 5

Sample size = n = 10

	CELL NUMBER (i) 1	2	3	4	5
mass value (h_i):	.2	.2	.2	.2	.2
cumulative mass value (H_i):	.2	.4	.6	.8	1.0
Number of observations (X_i):	3	3	4	0	0

b. Solution:

Read both sides of Card 1 into the calculator, then:

COMMENT	ENTER	PRESS	DISPLAY	COMMENT
initialize		E	0	H defined with h_i s
no. of cells	5	R/S	5	
sample size	10	R/S	1	cell number of next mass value entry
cell mass values	.2	R/S	2	
	.2	R/S	3	
	.2	R/S	4	
	.2	R/S	5	
	.2	R/S	1	cell number of next frequency count
observations per cell (frequency count)	3	R/S	2	
	3	R/S	3	
	4	R/S	4	
	0	R/S	5	
	0	R/S	0	
		A	.4	test statistic of two-sided test [Time: 22 sec]

Read both sides of Card 2 into the calculator, then:

PRESS	DISPLAY	COMMENT
A	.0414007042	point estimate of significance level [Time: 7 min]
B	.0416172032	upper bound on significance level
x:t	.0411842053	lower bound on significance level
C	.000216499	maximum error in point estimate
D	.0414007042	recall point estimate
E	479.59	return TI-59 to normal partitioning

The user can perform the same K-S test by initializing Card 1 with "E′" and then entering H_i values instead of h_i values. The test results will be the same.

2. Example 2

a. Problem:

H_0 : $F(x) \geq H(x)$ for all x, where H is defined below

H_1 : $F(x) < H(x)$ for some x

Number of cells = m = 3

Sample size = n = 15

	CELL NUMBER (i) 1	2	3
mass value (h_i):	.3624	.4167	.2209
cumulative mass value (H_i)	.3624	.7791	1.0
number of observations (X_i)	5	3	7

b. Solution:

Read both sides of Card 1 into the calculator, then:

COMMENT	ENTER	PRESS	DISPLAY	COMMENT
initialize		E′	0	H defined with H_i s
no.of cells	3	R/S	3	
sample size	15	R/S	1	cell number of next cumulative mass value
cumulative cell mass values	.3624	R/S	2	
	.7791	R/S	3	
	1.0	R/S	1	cell number of next frequency count
observations per cell (frequency counts)	5	R/S	2	
	3	R/S	3	
	7	R/S	0	
		B	.245766667	test statistic of one-sided test [Time: 15 sec]

Read both sides of Card 2 into the calculator, then:

PRESS	DISPLAY	COMMENT
A	.0395671995	significance level [Time: 11 min]
E	479.59	return TI-59 to normal partitioning

APPENDIX C

CONDENSED USER INSTRUCTIONS

The following two pages are condensed user instructions for Continuous K-S and Discrete K-S, respectively. It is recommended that the reader familiarize himself with the more expanded discussion of programs in sections IV and V above before relying on the condensed instructions.

The condensed instructions for both programs are written for a complete K-S test. It is possible to use Continuous K-S to evaluate a known test statistic from a continuous K-S test. The procedure is as follows.

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1.	Repartition TI-59	7	Op 17	399.69
2.	Enter both sides of Card 2 (Continuous K-S)			
3.	enter sample size	n	STO 59	n
4.	enter test statistic	t	STO 00	t
5.	compute significance level		E′	S.L.

If the test was one-sided the significance level computed is exact. If the test was two-sided the result should be doubled. The doubled value is an upper bound on the exact significance level.

CONTINUOUS K-S

Card 1 (banks 1 & 2): standard partitioning — Applied Statistics Module
Card 2 (banks 1 & 2): 399.69 partitioning — Printer optional

Purpose: K-S test on continuous random variable with unknown CDF, F. Maximum sample size n = 54. Direction of test specified by null hypothesis (H_0). Alternative hypothesis (H_1) is negation of H_0. Hypothesized CDF, H, can be uniform, normal, Weibull, or exponential. If another H is desired the user can add a subroutine transforming the desired H to one of the above distributions. Program computes exact significance level (S.L.) for a one-sided test and an upper bound on S.L. for two-sided test.

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY	PRINTOUT
1	Enter both sides of Card 1				
2.	Initialize		E	399.69	
3.	Enter sample data	X_i (i=1, 2,..., n)	A	i	X_i
4.	Sort data		C	1	$X_{(1)}, \ldots, X_{(n)}$

5. Tranforming data (optional step; normally omitted): If hypothesized CDF, H, is NOT uniform, normal, Weibull, or exponential then the user may enter a subroutine to transformed H to one of those distributions. The subroutine must begin with "LBL E′" and end with "INV SBR". For a transformation M the subroutine should be written to tranform some value c to M(c) with c in the x-register when the subroutine is called and M(c) is returned to the main program. Use only registers n+1 through 57 (sample size = n) and limit subroutine to 75 steps. After subroutine entered and validated, press "B′". $M(X_{(n)})$ will be displayed.
If user wishes to transform each sample value x to ln(x) press "A′". The transformation subroutine is already provided and is useful to transform lognormal (μ,σ^2) data to normal (μ,σ^2). Do not press "B′". Proceed with test with the new distribution substituted for original H.

STEP	PROCEDURE	ENTER	PRESS	DISPLAY	PRINTOUT
6.	Enter both sides of Card 2				
7.	Choose direction of test:				
	If H_0: $F(x) = H(x)$		A	0	
	If H_0: $F(x) \geq H(x)$		B	0	
	If H_0: $F(x) \leq H(x)$		C	0	
8.	Specify H for test:				
	If H = normal (μ,σ^2)	μ	A′	μ	
		σ	R/S	S.L.	S.L.
	If H = exponential (λ)	λ	B′	1	
		1	R/S	S.L.	S.L.
	If H = Weibull (λ, α)	λ	B′	1	
		α	R/S	S.L.	S.L.
	If H = uniform (a, b)	a	C′	a	
		b	R/S	S.L.	S.L.

9. The test statistic can be recalled from register 00.

DISCRETE K-S

Card 1 (banks 1 & 2): standard partitioning — Any Library Module
Card 2 (banks 1 & 2): 399.69 partitioning — Does not use printer

Purpose: K-S test on discrete random variable with unknown CDF, F. For sample size n and random variable with m possible outcomes (cells) the program will handle any problem with $n+m \leq 54$. Maximum number of cells: 20. If n+m slightly exceeds 54 program will attempt to solve; edit code "44" displayed if not possible. Program computes significance level of test.

Notation:
- X_i = number of observations of X in cell i; i = 1, . . . , m
- h_i = P(random observation is in cell i)
- H_i = P(random observation is in cell j; $j \leq i$)
- t = test statistic
- H_0 = null hypothesis
- H = hypothesized CDF of random variable X
- F = actual CDF of random variable X (unknown)

The hypothesized CDF, H, may be defined by specifying h_i or H_i values.

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1.	Initialize:			
	If H defined by h_i values		E	0
	If H defined by H_i values		E′	0
2.	Specify number of cells	m	R/S	m
3.	Specify sample size	n	R/S	1
4.	Define H with either h_i values or H_i values	h_i or H_i (i = 1, ..., m-1)	R/S	i+1
		h_m or H_m	R/S	1 (note 1)
2.	Specify cell frequency counts	X_i (i = 1, ... ,m-1)	R/S	i+1
		X_m	R/S	0 (note 2)
5.	Specify direction of test			
	If H_0: F(x) = H(x)		A	t
	If H_0: $F(x) \geq H(x)$		B	t
	If H_0: $F(x) \leq H(x)$		C	t (note 3)
6.	Enter both sides of Card 2			
7.	Compute significance level (S.L.)		A	S.L.
8.	Evaluate S.L. for two-sided test; (S.L. exact for one-sided test)		B	max S.L.
			x⇌t	min S.L.
			C	max error
			D	recall S.L.

Note 1: Edit code "22" indicates (h_1 + ... + h_m) greater than 1.01 or less than .99. User may press "A" to normalize h_i's & continue.
Note 2: Edit code "33" indicates (X_1 + ... + X_m) ≠ n. Cannot proceed.
Note 3: Edit code "44" indicates problem too large. Recall t from Reg 00.

APPENDIX D

USE OF PENDING ARITHMETIC REGISTERS IN THE TI-59

The TI-59 calculator has eight pending arithmetic registers. They are not accessible from the keyboard but are accessible from within a program and are used extensively in Discrete K-S. A detailed discussion of these special registers has been provided by a non-profit organization called the SR-52 Users Club [Ref. 10]. Unfortunately the user's manual provided by the manufacturer [Ref. 8] does not discuss them.

Each of the eight pending registers can be accessed with "82" followed by a two digit number, represented here as "mn". The access code "82" is represented as "HIR" by a Texas Instruments printer. This mnemonic has been interpreted as both "Hierarchy Internal Register" and "Hidden Internal Register".

The value of m determines which register operation is performed, i.e.,

m	operation
0	STO
1	RCL
3	SUM
4	Prd
5	INV SUM
6, 7, 8, or 9	INV Prd .

The value of n (1, 2, . . . , or 8) determines which HIR is

accessed. For example, to add a number directly to the value currently in HIR 4 the program codes "82, 34" would be used.

To enter a two digit code like "82" or "34" in a program it is convenient to "trick" the calulator with a register command. For instance, with the calculator in the "LRN" mode press "STO, 8, 2" then backstep (with "BST") twice and "42" (for "STO") will be displayed. Delete the register command "STO" by pressing "del" and the "82" remains as a merged code. Advance the program pointer with "SST" one step and the remaining program steps can be entered.

The pending arithmetic registers serve several purposes. When nested arithmetic operations are performed the operands are first pushed into HIR 1 and then into 2, 3, . . ., 8. Use of the Op codes 1, 2, 3, and 4 assign values to HIRs 5, 6, 7, and 8, respectively. Other keyboard operations which affect the HIRs are "P→R" and "D.MS" .

After extensive experimentation with HIRs the writer is convinced that the TI-59 uses HIRs in other ways besides those mentioned. For instance, Discrete K-S, Card 2, does not use nested arithmetic operations, "P→R", or "D.MS". It uses HIRs 2 - 7. When the program was modified to also use HIR 1 it did not perform computations correctly. For some reason the manufacturer has not disclosed any information about the HIRs except to acknowledge their existence.

Utilizing HIRs modestly increases the calculator's data storage capacity. It also provides a place to store a few

numbers to be retained after "CM" is pressed. When incorporating HIRs into a program the following approach is recommended. First, write the program without using any HIRs and validate it. Second, incorprate the desired HIRs to free regular data registers and then revalidate the program. Trying to find an error in a program that was written with HIRs initially can be very frustrating; the program may be affected by some mysterious HIR-related problem that the user may never find.

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